Book Review

Analysis of Shells and Plates, by Phillip L. Gould, Springer-Verlag, New York, 1988, 491 pp., $69.00.

The excellent preface by the author states his desire to unify the engineering theories of beams, plates, and shells under the heading of "Surface Structures," which, of course, implies going from the general to the special. The concomitant prerequisites for such an approach are not overlooked: at least an acquaintance with vector calculus, matrix algebra, and linear elasticity. For many graduate students the text is appropriate for a second level course in Civil, Mechanical, and Aerospace Engineering, and in Engineering Mechanics, although some undergraduate curricula may enable the more capable first-year graduate student to follow the developments. The material represents a significant enhancement of the author's 1974 book, bringing with it a reflection of subsequent years of practical and academic experiences throughout its 10 chapters.

After noting the difficulty in applying the theory of elasticity to engineering problems, the introduction briefly refers to beam, plate, and shell theories as being generally characterized by having at least one dimension, the thickness, small compared to the others. With appropriate reference to Navier, Bernoulli, Euler, Kirchhoff, and Love, the author meticulously describes the implications of this and other assumptions that form the foundations of these theories. Equally lucid is a discussion of the mode of load transmission in each of the structural models considered. By such efforts the author does a great service for future designers by emphasizing that load resistance in extensional modes is far more efficient than in flexural ones. The introduction closes with an inspiring set of photographs of famous shell structures that have withstood the devastation associated with time and usage. This first chapter, as well as all others, is concluded with suitable references and pertinent exercises. The remaining nine chapters clearly carry out the author's intention to present a solidly rigorous treatment of the subject material. His considerable thought and planning with the reader (student) in mind is commendable.

Although primarily concerned with structural analysis of shells, the text in Chapter 2 presents a brief, but comprehensive treatment of curved surfaces. The topical headings are: curvilinear coordinates, middle surface geometry, unit tangent vectors and principal directions, second quadratic form of the theory of surfaces, principal radii of curvature, Gauss-Codazzi relations, Gaussian curvature, and specialization of shell geometry (shallow shells, shells of revolution). To follow these developments the reader should be conversant with the vector calculus. Excellent exposition and illustrative diagrams afford a clear understanding of the various relations developed. The material here represents a reference to be used in subsequent chapters.

Equilibrium is the title of Chapter 3, which opens with a discussion of stress resultants and couples. For stresses, the text employs the notation and sign convention of elasticity theory, while for stress resultants and couples the conventions are those usually found in the engineering literature. With the use of principal orthogonal coordinates of Chapter 2, the integral expressions for the resultants in terms of stresses are developed consistent with linear theory. A discussion of the determination of approximate values for the stresses in thin shells once the resultants are known is also given. Next, a clear and concise derivation of the linear equations of equilibrium for a shell element is performed in terms of principal curvilinear coordinates. Vector algebra is applied throughout, with some discussion of the individual contributions.

The shell of revolution is then employed as a useful illustration of how to proceed from the general to the special, with diagrams clearly delineating individual contributions. Furthermore, a discussion of the physical significance of each term in the equations serves to make the results crystal clear. Of course, a derivation without the application of vector algebra would use such descriptions from the outset.

The general equilibrium equations for "medium-thin" plates are generated by allowing the principal radii to grow indefinitely large, and it is observed that the in-plane stress resultants become uncoupled from the stress couples and transverse shears, in contrast to those for shells.

Chapter 4 focuses on small-deformation membrane theory, again emphasizing the efficiency of in-plane stresses in resisting surface loads. Based upon usual assumptions, the general equilibrium equations are reduced to the three internally statically determinate equations of membrane theory. A clear discussion of the conditions under which such theory is applicable is then presented. The first example considered is the shell of revolution. The equilibrium equations are transformed using the auxiliary variables of Novozhilov, and the general axisymmetric problem is treated, including a special discussion of conditions relating to the pole. Spherical shell problems are then handled in detail: internal pressure, self-weight. The latter problem for hyperboloidal shells (negative Gaussian curvature), with emphasis on cooling towers, also receives detailed treatment, as does the toroidal shell under internal pressure. Solutions are also pursued on the basis of overall equilibrium considerations and include spherical, conical, and cylindrical shells as special cases. Attention is then directed to compound shells with emphasis on pressurized, capped circular cylindrical shells, including a discussion of the effects of discontinuities for various end closures (spherical, ellipsoidal, torospherical).

For nonsymmetric loading of shells of revolution the usual Fourier series approach is taken to reduce the problem to the solution of ordinary differential equations. The closed-form results of Novozhilov's solution for the first harmonic is then displayed, and the results for spherical shells under wind loads are depicted. Discussion is also devoted to rotational shells under seismic loading, in which the considerable experience of the author is brought to bear here as well as in subsequent sections.

Consideration of asymmetrical loading includes the closed-form solution of the basic membrane equations for spherical shells, with detailed application to such a shell supported on columns. Similar attention is given column-supported, bin-type cylindrical shells. This section concludes with a comprehensive treatment of circular cylindrical membrane shells including a comparison with beam theory, and consideration of such a shell under realistic lateral wind loading. Discussions pertaining to wind pressure on spherical and hyperboloidal shells, with a rather detailed analysis of the latter, are also presented. A commentary directs the reader's attention to the need for "computer-based numerical techniques" to solve most problems of asymmetrically loaded shells.

Analyses of membrane shells of translation emphasize open cylindrical shells (barrel vaults) as well as shells with double curvature, particularly hyperbolic paraboloidal forms. Solutions are presented with the use of the Pucher stress function and are applied to roof-type structures. The arch action of such shells is noted, as are the requirements for edge-support members. This chapter concludes with brief analyses of elliptic paraboloids and conoids.

Chapter 5 contains a comprehensive treatment of displacements, rotations, strains, curvatures, and twists for arbitrary shell geometry utilizing a vector approach. The final results are restricted to small deformations, while transverse shearing strains are retained in the general relations. This reviewer was a little bothered by the use of D_α, D_β, D_n rather than u, v, w or u_α, u_β, u_n for translations, and by $D_{\alpha\beta}$, $D_{\beta\alpha}$ for rotations (ω_β, ω_α). Final expressions reflect the usual assumptions that the thickness is small compared to the local radii of curvature. As an example, the general relations are specialized to the shell of revolution (with and without transverse shearing strains), together with relevant physical interpretations. A second example yields relations for flat plates, clearly revealing the complications arising when transverse shearing strains are retained.

Chapter 6 discusses constitutive laws, including forms suitable for rather general material representations. Attention is given to orthotropic materials, including the concept of equivalent thickness as it applies to stiffened shells and plates, and to reticulated shells. Boundary conditions are also treated with the utmost clarity, including those associated with Kirchhoff. Special consideration is afforded membrane theory displacements and their primary role in determining solutions to the complete shell problem. Detailed analyses are then developed for both the axisymmetric and nonsymmetric deformation of shells of revolution, with it being noted that numerical techniques are generally required for the latter, as well as for other shells with double curvature. The determination of displacements for circular cylindrical membrane shells is also considered in this chapter.

A brief discussion of energy methods is given in Chapter 7. Topics treated include strain energy, potential of external loads, principles of virtual displacements and of minimum (stationary) total potential energy, and Rayleigh-Ritz and Galerkin methods. The variational operator δ is introduced but its role in the calculus of variations is not elaborated upon. The relevance of the Rayleigh-Ritz technique to the finite-element method is also indicated.

A comprehensive treatment of plate bending theory is presented in Chapter 8, based upon the general relations developed in previous chapters. Although the procedure for the determination of the final classical equations in general coordinates is described, details are restricted to the use of Cartesian and polar coordinates in the development of the equilibrium equations, boundary conditions, and associated relations.

After considering rectangular plates under several combinations of uniformly distributed edge moments, the text discusses the Navier and Levy-Nadai solutions, including several detailed analyses. Solutions are then described for circular plates subjected to axisymmetric deformations. These analyses are followed by a presentation of the homogeneous solution for the general bending equation, which is then applied toward the solution of the linearly varying load problem. Brief discussions for triangular and elliptical plates, and for circular sector plates follow, and the efficacy of the finite-element technique for solving plate problems with irregular boundaries is mentioned.

Strain energy expressions are then determined in Cartesian as well as polar coordinates, including thermal effects. The principle of virtual work is subsequently applied to a simply supported rectangular plate under a concentrated load, as is the Rayleigh-Ritz method to a uniformly loaded clamped plate. Subsequent application of the Galerkin method to the latter problem utilizing the same shape function yields, of course, identical results. Other topics briefly treated are variable flexural rigidity, and orthotropic and multilayered plates. A very useful form for strain energy, including effects of transverse shearing deformations, is also derived from earlier results, and an interesting discussion of folded plate theory is presented.

The latter part of Chapter 8 is comprised of topics pertinent to instability and finite deformation of plates, with the previously described differential equations and associated strain energies being modified. The usual differential equation solution for a simply-supported rectangular plate under transverse and unidirectional in-plane loadings is described, including a discussion of buckling.

Application of the virtual work principle, together with a Fourier series representation of the deflected shape, also yields a solution to the simply-supported rectangular plate, now under a transverse concentrated load and uniform bidirectional in-plane loadings. Concluding sections of this chapter develop the von Kármán equations in rectangular coordinates, as well as the axisymmetric large-deflection equations for circular plates. The former are applied to cylindrical bending, while the latter are solved for uniform-edge moments. Appropriate discussions of the results are included.

Chapter 9 returns to shells with specializations of the general relations derived in previous chapters. Special emphasis is placed upon circular cylindrical shells, with the resulting classical equations being equivalent to those of Eric Reissner (1941). Solutions are determined for several axisymmetric problems involving edge bending and transverse shear loads, circumferential line loads, normal surface loads, as well as the effect of temperature gradients across the shell wall. The effects of stiffening rings are also investigated. The relations applied to the general loading problem are essentially those of Donnell, so that a single uncoupled eighth-order equation is found for the normal displacement component, while two fourth-order equations each coupled only with this component completes the set. Fourier series solutions to the homogeneous equations for both complete and open shells are described, as are those for column-supported vertical cylindrical shells and multiple barrel roof-type structures.

Because of the effectiveness of finite element solutions in analyzing general problems of shells of revolution, lengthy treatment of well-known classical solutions is omitted, and relevant references are listed. A discussion of shells of revolution starts with a derivation of the axisymmetric equilibrium equations for the deformations of an orthotropic shell, with and without the effects of transverse shearing strains. These equations are then specialized to apply to edge loaded isotropic spherical shells, so that solutions of the resulting hypergeometric equation can be used in combination with solutions of surface loaded spherical membrane shells. Asymptotic solutions are briefly discussed, as is the related simplification of the shell equations leading to the Geckeler approximation, which is essentially a solution to an equivalent circular cylindrical shell problem.

Next, the analysis of a cylindrical pressure vessel having a spherical shell closure is formulated in detail, with the use of matrices facilitating the discussion. The interface discontinuities and their amelioration through the use of appropriate structural elements are discussed. This section of Chapter 9 also refers to the general solutions for asymmetrically loaded shells of revolution developed by Flugge and by Novozhilov. In view of the difficulties of such solutions, the finite-element technique, based on the Rayleigh-Ritz method, together with Fourier series representations of stress and deformation variables in the circumferential direction and polynomials along meridians, is outlined in considerable detail. A discussion of the Mushtari-Donnell-Vlasov theory for shells of translation is also presented, followed by the closely related theory of shallow shells, including the corresponding relatively simple strain energy expression.

The final sections of this chapter begin with a lucid description of the various forms of small- and large-deflection elastic collapse phenomena. Derivations are then given for the differential equations corresponding to the Mushtari-Donnell-Vlasov theory for buckling, and for the related strain energy expression for the axisymmetric buckling of an axially compressed circular cylindrical shell. Subsequently, a Rayleigh-Ritz procedure leads to the classical buckling load. Additionally, Vlasov's solution of the shallow shell equations for a spherical shell under external pressure yields the corresponding classical buckling load.

Vlasov's equations for the finite deformation of shallow shells are also developed, and a discussion of the energy solution of von Kármán and Tsien for circular cylindrical shells is presented together with reference to the inadequacy of the classical buckling load. This discussion is followed by a concise, but quite useful, description of the effects of initial imperfections on the elastic collapse load of shells of revolution under external lateral pressure.

In the final four pages (Chapter 10) of this excellent text. Professor Gould discusses the many facets of shell design, including the role of reinforcing components.

I highly recommend this very readable book to engineers and students who wish to delve rather deeply into the theory and application of shell and plate structures.

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